QUEEN'S UNIVERSITY

Faculty of Engineering and Applied Science

Department of Mechanical Engineering

MECH 448

COMPRESSIBLE FLUID FLOW

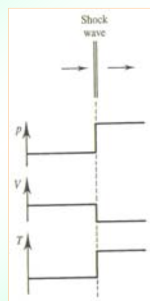
2011

Chapter Five

NORMAL SHOCK WAVES

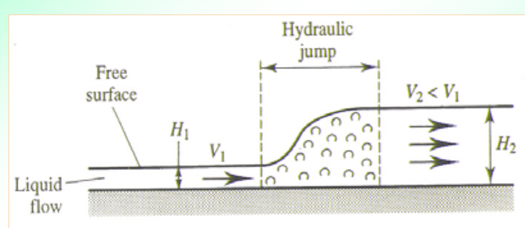
SHOCK WAVES: It has been found experimentally that, under some circumstances, it is possible for an almost spontaneous change to occur in a flow, the velocity decreasing and the pressure increasing through this region of sharp change. The possibility that such a change can occur actually follows from the analysis given below. It has been found experimentally, and it also follows from the analysis given below, that such regions of sharp change can only occur if the initial flow is supersonic. The extremely thin region in which the transition from the supersonic velocity, relatively low pressure state to the state that involves a relatively low velocity and high pressure is termed a shock wave.

The changes that occur through a normal shock wave, i.e., a shock wave which is straight with the flow at right angles to the wave, is shown in the following figure:



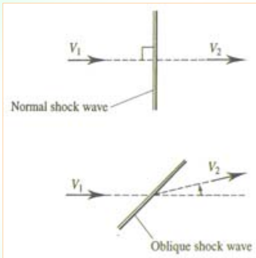
A photograph of a normal shock wave is shown in the following figure:

A shock wave is extremely thin, the shock wave normally only being a few mean free paths thick. A shock-wave is analogous in many ways to a “hydraulic-jump” that occurs in free-surface liquid flows, a hydraulic jump being shown schematically below. A hydraulic jump occurs, for example, in the flow downstream of a weir.



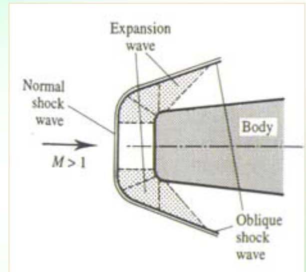
Mech 448

In the case of a normal shock wave, the velocities both ahead (i.e. upstream) of the shock and after (i.e., downstream) of the shock are at right angles to the shock wave. In the case of an oblique shock wave there is a change in flow direction across the shock. This is illustrated in the following figure:



Mech 448

A complete shock wave may be effectively normal in part of the flow, curved in other parts of the flow and effectively oblique in other parts of the flow as shown in the following figure:



Mech 448

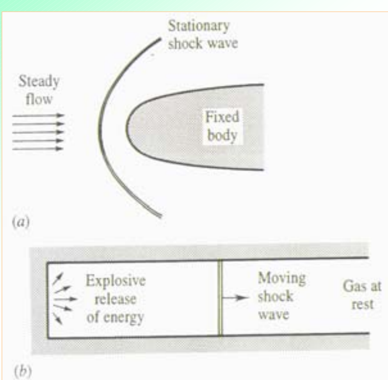
Because of their own importance and because, as will be shown later, the oblique shock relations can be deduced from those for a normal shock wave, the normal shock wave will be first be considered in the present chapter. Oblique shock waves will then be discussed in the next chapter. Curved shock waves are relatively difficult to analyze and they will not be discussed in detail in the present course.

Normal shock waves occur, for example, in the intakes to the engines in some supersonic aircraft, in the exhaust system of reciprocating engines, in long distance gas pipe-lines and in mine shafts as a result of the use of explosives.

Mech 448

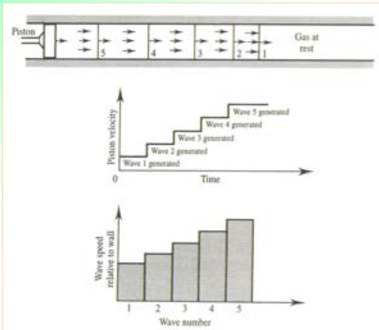
When a normal shock wave occurs, for example, in a steady flow through duct, it can be stationary with respect to the coordinate system which is fixed relative to the walls of the duct. Such a shock wave is called a stationary shock wave since it is not moving relative to the coordinate system used. On the other hand, when a sudden disturbance occurs in a flow, such as, for example, the sudden closing of a valve in a pipe-line or an explosive release of energy at a point in a duct, a normal shock wave can be generated which is moving relative to the duct walls. This is illustrated in the following figure.

Mech 448



Mech 448

To illustrate how a shock wave can form, consider the generation of a sound wave as discussed earlier. It was assumed that there was a long duct containing a gas at rest with a piston at one end of this duct that was initially at rest. Then, at time 0, the piston was given a small velocity into the duct giving rise to a weak pressure pulse, i.e., a sound wave, that propagated down the duct (see following figure).



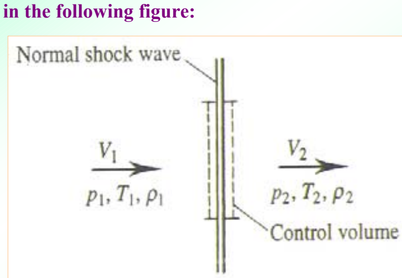
Generation of a Normal Shock Wave

If dV is the velocity given to the piston, which is, of course, the same as the velocity of the gas behind the wave, then the increase in pressure and temperature behind the wave are equal to $\rho\ a\ dV$ and [(γ - 1) $T\ dV$ / a] respectively. Since ρ, a, and T are all positive, this shows that the pressure and temperature both increase across the wave. It was also shown that the velocity at which the wave moves down the duct is equal to $\sqrt{\gamma RT}$, which is by definition the speed of sound. Therefore, since the temperature increases across the wave, the speed of sound behind the wave will be $a + da$, where da is positive. Now consider what happens if some time after the piston is given velocity dV into the duct, its velocity is suddenly again increased to 2 dV. As a result of the second increase in piston speed, a second weak pressure wave will be generated that follows the first wave down the duct as shown in the above figure.

This second wave will be moving relative to the gas ahead of it at the speed of sound in the gas through which it is propagating. But the gas ahead of the second wave has velocity dV. Hence, the second wave moves relative to the duct at a velocity of $a + da + dV$. But, the first wave is moving at a velocity of a relative to the duct. Therefore, since both da and dV are positive, the second wave is moving faster than the first wave and, if the duct is long enough, the second wave will overtake the first wave. But the second wave cannot pass through the first wave. Instead, the two waves merge into a single stronger wave. If, therefore, the piston is given a whole series of step increases in velocity, a series of weak pressure waves will be generated which will all eventually overtake each other and merge into a single strong wave if the duct is long enough, i.e., a moving normal shock wave will be generated.

The analysis of stationary normal shock waves will first be considered and then the application of this analysis to moving normal shock waves will then be discussed.

STATIONARY NORMAL SHOCK WAVES: Attention will first be given to the changes that occur though a stationary normal shock wave. In order to analyze the flow though a stationary normal shock wave, consider a control volume of the form indicated in the following figure:



This control volume has a cross sectional area of A normal to the flow direction. The shock wave relations are obtained by applying the laws of conservation of mass, momentum, and energy to this control volume. Conservation of mass gives:

$$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A$$

i.e.:

$$\rho_1 V_1 = \rho_2 V_2$$

Mech 448

Next consider conservation of momentum. Since the only forces acting on the control volume in the flow direction are the pressure forces, conservation of momentum applied to the control volume gives:

$$p_1 - p_2 = \rho_1 V_1 (V_2 - V_1)$$

or

$$p_1 - p_2 = \rho_2 V_2 (V_2 - V_1)$$

Mech 448

These two equations can be rearranged to give:

$$V_1 V_2 - V_1^2 = \frac{p_1 - p_2}{\rho_1}$$

and:

$$V_2^2 - V_2 V_1 = \frac{p_1 - p_2}{\rho_2}$$

Adding these two equations together then gives:

$$V_2^2 - V_1^2 = (p_1 - p_2)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

Mech 448

Lastly, consider the application of conservation of energy to the flow across the shock wave. Because one-dimensional flow is being considered there are no changes in the flow properties in any direction that is normal to that of the flow and, because the upstream and downstream faces of the control volume lie upstream and downstream of the shock wave, there are no temperature gradients normal to any face of the control volume. The flow through the control volume is, therefore, adiabatic and the energy equation, therefore, gives:

$$\frac{V_1^2}{2} + c_p T_1 = \frac{V_2^2}{2} + c_p T_2 = c_p T_0 = \text{constant}$$

The stagnation temperature therefore does not change across the shock.

Mech 448

Applying the equation of state across the wave gives:

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$

The above four equations obtained by applying conservation of mass, conservation of momentum, conservation of energy and the equation of state can be combined to give the following:

Mech 448

$$\frac{\rho_2}{\rho_1} = \frac{\left[\left(\frac{\gamma+1}{\gamma-1}\right)\frac{p_2}{p_1} + 1\right]}{\left[\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{p_2}{p_1}\right]} \qquad \frac{V_1}{V_2} = \frac{\left[\left(\frac{\gamma+1}{\gamma-1}\right)\frac{p_2}{p_1} + 1\right]}{\left[\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{p_2}{p_1}\right]}$$

$$\frac{T_2}{T_1} = \frac{\left[\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{p_2}{p_1}\right]}{\left[\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{p_1}{p_2}\right]}$$

Mech 448

The above three equations give the density, velocity and temperature ratios, ρ_2/ρ_1, V_2/V_1, and T_2/T_1, across a normal shock wave in terms of the pressure ratio, p_2/p_1, across the shock wave. The pressure ratio, p_2/p_1, is often termed the strength of the shock wave. This set of equations is often termed the Rankine-Hugoniot normal shock wave relations.

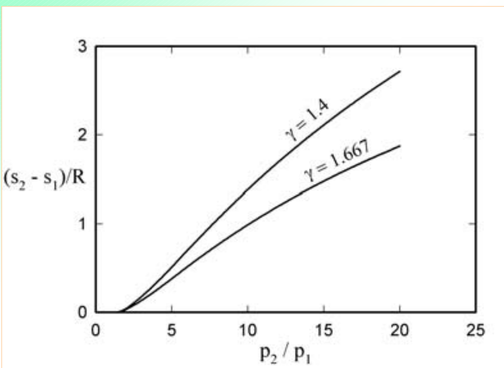
While the application of conservation of mass, momentum and energy principles shows that a shock wave can exist, it does not indicate whether the shock can be either compressive (i.e., $p_2 / p_1 > 1$) or expansive (i.e., $p_2 / p_1 < 1$). To examine this, the second law of thermodynamics must be used. Now the entropy change across the shock wave is given by:

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

Using the relations for T_2 / T_1 and p_2 / p_1 given above then gives:

$$\frac{s_2 - s_1}{R} = \ln\left\{\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma-1}}\left[\frac{(\gamma+1)\dfrac{p_2}{p_1} + (\gamma-1)}{(\gamma+1) + (\gamma-1)\dfrac{p_2}{p_1}}\right]^{\frac{-\gamma}{\gamma-1}}\right\}$$

The variations of (s_2 - s_1) / R for various values of γ (γ is always greater than 1) as given by the above equation are shown in the following figure:



Now, the second law of thermodynamics requires that for an adiabatic process the entropy must remain unchanged or must increase, i.e., it requires that:

$$\frac{s_2 - s_1}{R} \geq 0$$

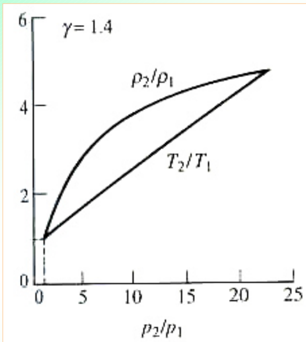
Using the above equation for the entropy change, or the figure given above, it is found that this will only occur if:

$$\frac{p_2}{p_1} \geq 0$$

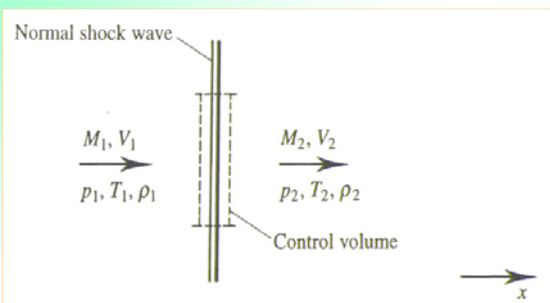
It therefore follows that the shock wave must always be compressive, i.e., that p_2 / p_1 must be greater than 1, i.e., the pressure must always increase across the shock wave. Using the equations for the changes across a normal shock then shows that that the density always increases, the velocity always decreases and the temperature always increases across a shock wave.

The entropy increase across the shock is, basically, the result of the fact that, because the shock wave is very thin, the gradients of velocity and temperature in the shock are very high. As a result, the effects of viscosity and heat conduction are important within the shock leading to the entropy increase across the shock wave.

Because the flow across a shock is adiabatic, the stagnation temperature does not change across a shock wave. However, because of the entropy increase across a shock, the stagnation pressure always decreases across a shock wave.



NORMAL SHOCK WAVE RELATIONS IN TERMS OF MACH NUMBER: While the relations derived in the previous section for the changes across a normal shock in terms of the pressure ratio across the shock, i.e., in terms of the shock strength, are the most useful form of the normal shock wave relations for some purposes, it is often more convenient to have these relations in terms of the upstream Mach number M_1. To obtain these forms of the normal shock wave relations, it is convenient to start again with a control volume across the shock wave such as that shown in the following figure and to again apply conservation of mass, momentum and energy to this control volume but in this case to rearrange the resulting relations in terms of Mach number.



In writing the conservation laws, no generality is lost by taking the area of the control volume parallel to the wave as unity. Conservation of mass then gives:

$$\rho_1 V_1 = \rho_2 V_2$$

Dividing this equation by a_1 then gives:

$$\rho_1 \frac{V_1}{a_1} = \rho_2 \frac{V_2}{a_2}\frac{a_2}{a_1}$$

which can be rewritten in terms of Mach numbers as:

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\frac{a_1}{a_2}$$

Next consider conservation of momentum applied to the control volume shown earlier. This gives:

$$p_1 - p_2 = \rho_2 V_2^2 - \rho_1 V_1^2$$

Hence, since:

$$a^2 = \frac{\gamma p}{\rho} \quad \text{i.e.,} \quad p = \frac{a^2 \rho}{\gamma}$$

the above equation gives:

$$\frac{\rho_2}{\rho_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\left(\frac{a_2}{a_1}\right)$$

Lastly, consider the application of the conservation of energy principle to the control volume used above. This gives:

$$V_1^2 + \left(\frac{2}{\gamma - 1}\right)a_1^2 = V_2^2 + \left(\frac{2}{\gamma - 1}\right)a_2^2$$

Dividing this equation by $2a_1^2/(\gamma - 1)$ gives on rearrangement:

$$\left(\frac{a_2}{a_1}\right) = \left[\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right]$$

Mech 448

Combining the equations given above leads on rearrangement to:

$$M_2^2 = \frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)} \qquad \frac{p_2}{p_1} = \frac{2\gamma M_1^2-(\gamma-1)}{(\gamma+1)}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$\frac{T_2}{T_1} = \left(\frac{a_2}{a_1}\right) = \frac{\left[2\gamma M_1^2-(\gamma-1)\right]\left[2+(\gamma-1)M_1^2\right]}{(\gamma+1)^2 M_1^2}$$

Mech 448

The stagnation pressure ratio across a normal shock wave is obtained by noting that:

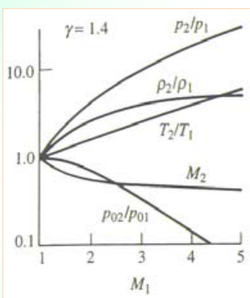
$$\frac{p_0}{p} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} \quad \text{and} \quad \frac{p_{02}}{p_{01}} = \left(\frac{p_{02}/p_2}{p_{01}/p_1}\right)\frac{p_2}{p_1}$$

From which it follows that:

$$\frac{p_{02}}{p_{01}} = \left[\frac{\gamma-1}{2}\frac{M_1^2}{\left(1+\frac{\gamma-1}{2}M_1^2\right)}\right]^{\gamma/(\gamma-1)} \left[\left(\frac{2\gamma}{\gamma+1}\right)M_1^2-\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{-1/(\gamma-1)}$$

Mech 448

The variations of pressure ratio, density ratio, temperature ratio and downstream Mach number with upstream Mach number given by these equations are shown in the following figure for the case of γ =1.4:



Mech 448

Consideration will lastly again be given to the change in entropy across a normal shock wave in terms of the upstream Mach number. Now it was shown previously that the change in entropy is given by:

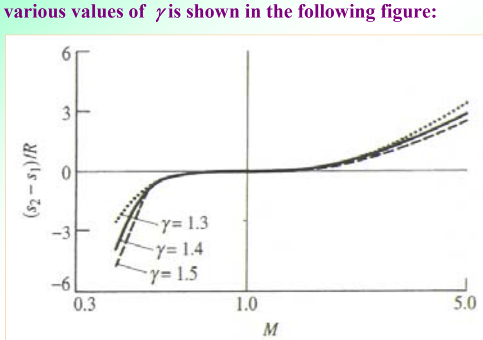
$$\frac{s_2-s_1}{R} = \ln\left[\left(\frac{p_2}{p_1}\right)^{1/(\gamma-1)}\left(\frac{\rho_2}{\rho_1}\right)^{-\gamma/(\gamma-1)}\right]$$

The right hand side of this equation can be expressed in terms of the upstream Mach number by using the relationships derived above for the pressure and density ratios. Using these gives:

$$\frac{s_2-s_1}{R} = \ln\left\{\left[\frac{2\gamma}{\gamma+1}(M_1^2-1)+1\right]^{1/(\gamma-1)}\left[\frac{(\gamma-1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{-\gamma/(\gamma-1)}\right\}$$

Mech 448

The variation of $(s_2 - s_1)/R$ with M_1 as given by this equation for various values of γ is shown in the following figure:



Mech 448

Now as discussed before , the second law of thermodynamics requires that for an adiabatic process the entropy must remain unchanged or must increase, i.e., it requires that:

$$\frac{s_2-s_1}{R} \geq 0$$

It will be seen from the results given in the above figure that for this to be satisfied it is necessary that:

$$M_1 \geq 1$$

It, therefore, follows that the Mach number ahead of a shock wave must always be greater than 1 and that the shock wave must, therefore, as discussed before, always be compressive, i.e., the pressure must always increase across the shock wave.

It then follows from:

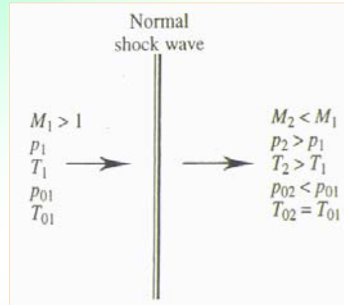
$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

that:

$$M_2 \le 1$$

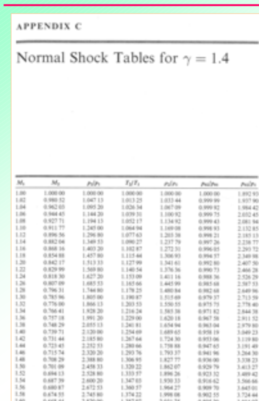
i.e., the flow downstream of a normal shock wave will always be subsonic.

These conclusions about the changes across a normal shock wave are summarized in the following figure:



NORMAL SHOCK WAVE TABLES: A number of sets of tables and graphs are available which list the ratios of the various flow variables such as pressure, temperature and density across a normal shock wave and the downstream Mach number as a function of the upstream Mach number for various gases, i.e. for various values of γ . An example of a set of tables is shown below.

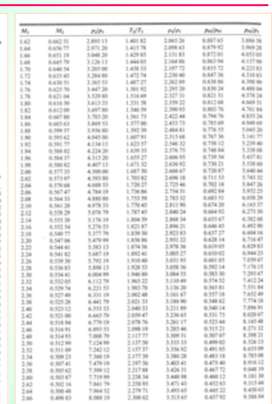
The values in these tables and graphs are, of course, derived using the equations given in the previous section. As with isentropic flow, instead of using tables, it is often more convenient to use a computer program to find the changes across a shock wave. Alternatively, most calculators can be easily programmed to give results for a normal shock wave.



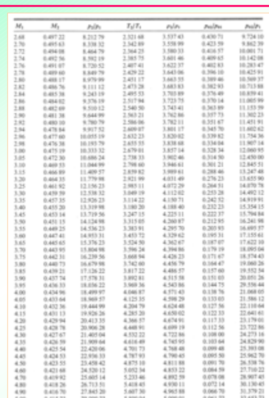
APPENDIX C

Normal Shock Tables for $\gamma = 1.4$

M_1	M_2	p_2/p_1	T_2/T_1	ρ_2/ρ_1	p_{02}/p_{01}	p_{02}/p_1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

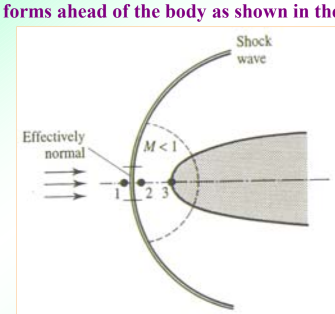


M_1	M_2	p_2/p_1	T_2/T_1	ρ_2/ρ_1	p_{02}/p_{01}	p_{02}/p_1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



M_1	M_2	p_2/p_1	T_2/T_1	ρ_2/ρ_1	p_{02}/p_{01}	p_{02}/p_1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

THE PITOT TUBE IN SUPERSONIC FLOW: Consider flow near the front of a blunt body placed in a supersonic flow as shown in the following figure. Because the flow is supersonic, a shock wave forms ahead of the body as shown in the figure.

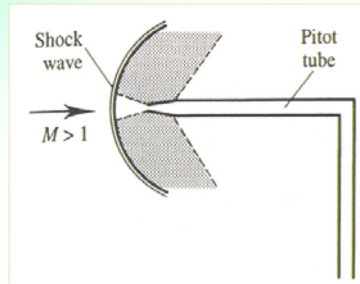


The shock wave ahead of the body is curved in general but ahead of the very front of the body, the shock is effectively normal to the flow. Hence, the conditions across the shock, i.e., between points 1 and 2 in the above figure, are related by the normal shock relations. Further, since the flow downstream of a normal shock wave is always subsonic, the deceleration from point 2 in the figure to point 3 in this figure, where the velocity is effectively zero can, as discussed in the previous chapter, be assumed to be an isentropic process. Using this model of the flow, the pressure at the stagnation pressure can be calculated for any specified upstream conditions.

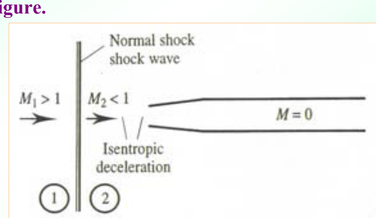
The flow model is thus:

Ahead of 1	-	Undisturbed Flow
1 to 2	-	Normal Shock Wave
2 to 3	-	Isentropic Deceleration to M=0

When a Pitot tube is placed in a supersonic flow, a type of flow similar to that indicated in the above figure occurs i.e the flow over a Pitot tube in supersonic flow resembles that shown below:



Since there will be a change in stagnation pressure across the shock wave, it is not possible to use the subsonic pitot tube equation in supersonic flow. However, as noted above, over the small area of the flow covered by the pressure tap in the nose of the pitot tube the shock wave is effectively normal and the flow behind this portion of the shock wave is, therefore, subsonic and the deceleration isentropic, these assumptions being shown in the following figure.



The flow can, therefore, be analyzed as follows:

1. The pressure ratio across the shock wave, p_2 / p_1, can be found using normal shock wave relations,

2. The pressure at the stagnation point can be found by assuming that the isentropic relations apply between the flow behind the shock and the stagnation point.

Hence, since:

$$\frac{p_{02}}{p_1} = \frac{p_{02}}{p_2}\frac{p_2}{p_1}$$

where the subscripts 1 and 2 denote the conditions upstream and downstream of the shock wave respectively, using the relations previously derived, this equation gives:

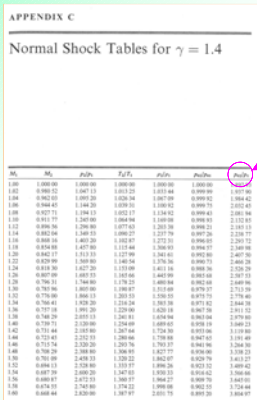
$$\frac{p_{02}}{p_1} = \left[1+\frac{\gamma-1}{2}M_2^2\right]^{\gamma/(\gamma-1)}\left[\frac{2\gamma M_1^2-(\gamma-1)}{\gamma+1}\right]$$

Therefore, using the expression for the downstream Mach number, it follows, on rearrangement, that:

$$\frac{p_{02}}{p_1} = \frac{\left[\dfrac{(\gamma+1)M_1^2}{2}\right]^{\gamma/(\gamma-1)}}{\left[\left(\dfrac{2\gamma M_1^2}{\gamma+1}\right)-\left(\dfrac{\gamma-1}{\gamma+1}\right)\right]^{1/(\gamma-1)}}$$

This equation is known as the Rayleigh Supersonic Pitot Tube equation. If p_{02} and p_1 are measured, this equation allows M_1 to be found. The value of p_{02} / p_1 is usually listed in normal shock wave tables or given by software.

Mech 448



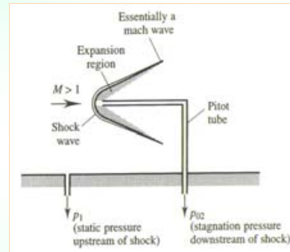
APPENDIX C

Normal Shock Tables for $\gamma = 1.4$

M_1	M_2	p_2/p_1	T_2/T_1	ρ_2/ρ_1	p_{02}/p_{01}	p_{02}/p_1
1.00	1.000 00	1.000 00	1.000 00	1.000 00	1.000 00	1.892 93
1.02	0.980 52	1.047 13	1.013 25	1.033 44	0.999 99	1.937 90
1.04	0.962 03	1.095 20	1.026 34	1.067 09	0.999 92	1.984 42
1.06	0.944 45	1.144 20	1.039 31	1.100 92	0.999 75	2.032 45
1.08	0.927 71	1.194 13	1.052 17	1.134 92	0.999 43	2.081 94
1.10	0.911 77	1.245 00	1.064 94	1.169 08	0.998 93	2.132 85
1.12	0.896 56	1.296 80	1.077 63	1.203 38	0.998 21	2.185 13
1.14	0.882 04	1.349 53	1.090 27	1.237 79	0.997 26	2.238 77
1.16	0.868 16	1.403 20	1.102 87	1.272 31	0.996 05	2.293 72
1.18	0.854 88	1.457 80	1.115 44	1.306 93	0.994 57	2.349 98
1.20	0.842 17	1.513 33	1.127 99	1.341 61	0.992 80	2.407 50
1.22	0.829 99	1.569 80	1.140 54	1.376 36	0.990 73	2.466 28
1.24	0.818 30	1.627 20	1.153 09	1.411 16	0.988 36	2.526 29
1.26	0.807 09	1.685 53	1.165 66	1.445 99	0.985 68	2.587 53
1.28	0.796 31	1.744 80	1.178 25	1.480 84	0.982 68	2.649 96
1.30	0.785 96	1.805 00	1.190 87	1.515 69	0.979 37	2.713 59
1.32	0.776 00	1.866 13	1.203 53	1.550 55	0.975 75	2.778 40
1.34	0.766 41	1.928 20	1.216 24	1.585 38	0.971 82	2.844 38
1.36	0.757 18	1.991 20	1.229 00	1.620 18	0.967 58	2.911 52
1.38	0.748 29	2.055 13	1.241 81	1.654 94	0.963 04	2.979 80
1.40	0.739 71	2.120 00	1.254 69	1.689 65	0.958 19	3.049 23
1.42	0.731 44	2.185 80	1.267 64	1.724 30	0.953 06	3.119 80
1.44	0.723 45	2.252 53	1.280 66	1.758 88	0.947 65	3.191 49
1.46	0.715 74	2.320 20	1.293 76	1.793 37	0.941 96	3.264 30
1.48	0.708 29	2.388 80	1.306 95	1.827 77	0.936 00	3.338 23
1.50	0.701 09	2.458 33	1.320 22	1.862 07	0.929 79	3.413 27
1.52	0.694 13	2.528 80	1.333 57	1.896 26	0.923 32	3.489 42
1.54	0.687 39	2.600 20	1.347 03	1.930 33	0.916 62	3.566 66
1.56	0.680 87	2.672 53	1.360 57	1.964 27	0.909 70	3.645 01
1.58	0.674 55	2.745 80	1.374 22	1.998 08	0.902 55	3.724 44
1.60	0.668 44	2.820 00	1.387 97	2.031 75	0.895 20	3.804 97

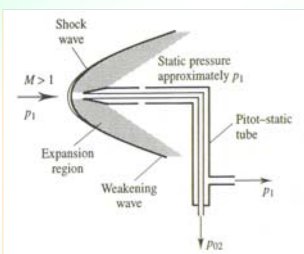
Mech 448

It should be noted that the static pressure ahead of the shock wave, i.e., p_1 , must be measured. If the flow is very nearly parallel to a plane wall there will be essentially no static pressure changes normal to the flow direction and p_1 can then be found using a static hole in the wall as indicated in the following figure:



Mech 448

However, it has been found that a Pitot-static tube can also be used in supersonic flow because the shock wave interacts with the expansion waves (see later) decaying rapidly to a Mach wave and the pressure downstream of the vicinity of the nose of the Pitot tube is thus essentially equal to p_1 again as indicated in the following figure.

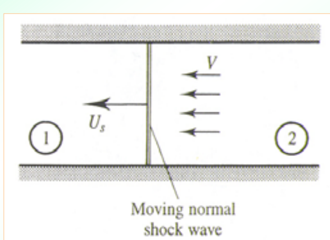


Mech 448

MOVING NORMAL SHOCK WAVES: In the above discussion of normal shock waves, the coordinate system was so chosen that the shock wave was at rest. In many cases, however, it is necessary to derive results for the case where the shock wave is moving relative to the coordinate system.

Mech 448

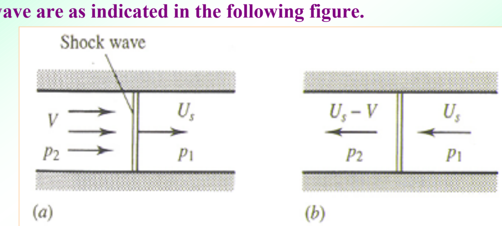
Consider the case where the gas ahead of the shock wave is stationary with respect to the coordinate system chosen and where the normal shock wave is moving into this stationary gas inducing a velocity in the direction of shock motion as indicated in the following figure.



Mech 448

Such moving shock waves occur, for example, in the inlet and exhaust systems of I.C. engines, in air-compressors, as the result of explosions and in pipe-lines following the opening or closing of a valve.

The required results can be obtained from those that were derived above for a stationary normal shock wave by noting that the velocities relative to a coordinate system fixed to the shock wave are as indicated in the following figure.



Hence, it follows that:

$$V_1 = U_S \quad , \quad V_2 = U_S - V$$

Since the direction of the flow is obvious, only the magnitudes of the velocities will be considered here. The Mach numbers upstream and downstream of the shock wave relative to the shock wave are given by:

$$M_1 = \frac{U_S}{a_1} = M_S$$

$$M_2 = \frac{U_S}{a_2} - \frac{V}{a_2} = \frac{U_S}{a_1}\frac{a_1}{a_2} - \frac{V}{a_2} = M_S \frac{a_1}{a_2} - \frac{V}{a_2} = M_S \frac{a_1}{a_2} - M_2'$$

where:

$$M_S = \frac{U_s}{a_1} \quad \text{and} \quad M_2' = \frac{V}{a_2}$$

M_s is the "shock Mach number". Substituting the value of M_1 for the moving shock into the equations previously given for a stationary normal shock wave then gives:

$$\frac{p_2}{p_1} = \frac{2\gamma M_S^2 - (\gamma - 1)}{(\gamma + 1)} \qquad \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_S^2}{2 + (\gamma - 1) M_S^2}$$

$$\frac{T_2}{T_1} = \left(\frac{a_2}{a_1}\right) = \frac{\left[2\gamma M_S^2 - (\gamma - 1)\right]\left[2 + (\gamma - 1) M_S^2\right]}{(\gamma + 1)^2 M_S^2}$$

$$M'^2 = \frac{(\gamma - 1) M_S^2 + 2}{2\gamma M_S^2 - (\gamma - 1)}$$

Normal shock wave tables and software can be used to evaluate the properties of a moving normal shock wave. To do this, M_1 is set equal to M_s and the tables or software are then used to directly find the pressure, density and temperature ratios across the moving shock wave. Further, since:

$$M_2' = M_S \frac{a_1}{a_2} - M_2$$

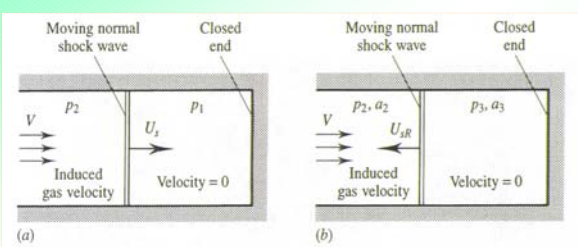
and since M_2 is given by the normal shock tables or software, M'_2, can be found.

Also since:

$$\frac{V}{a_1} = M_2' \frac{a_2}{a_1} = M_S - M_2 \frac{a_2}{a_1}$$

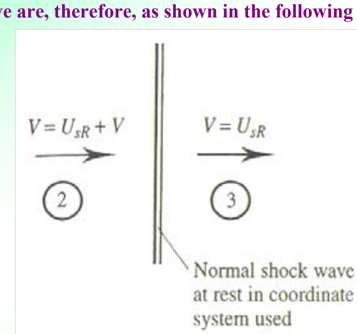
V can also thus be deduced using normal shock tables or software.

Brief consideration will now be given to the "reflection" of a moving shock wave off the closed end of a duct. Consider a moving normal shock wave propagating into a gas at rest in a duct. The shock, as discussed above, induces a flow behind it in the direction of shock motion. If the end of the duct is closed, however, there can be no flow out of the duct, i.e., the velocity of the gas in contact with the closed end must always be zero. Therefore, a normal shock wave must be "reflected" off the closed end, the strength of this "reflected" shock wave being just sufficient to reduce the velocity to zero. This is illustrated in the following figure.



Reflection of a moving normal shock wave from the closed end of a duct.

Consider a set of coordinates attached to the reflected normal shock wave. The gas velocities relative to this reflected shock wave are, therefore, as shown in the following figure.



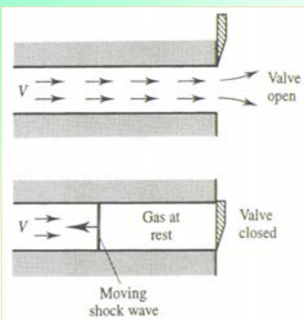
Now:

$$M_{R1} = \frac{U_{SR} + V}{a_2} = M_{SR} + M_2'$$

$$M_{R2} = \frac{U_{SR}}{a_3} = \left(\frac{U_{SR}}{a_2}\right)\left(\frac{a_2}{a_3}\right) = M_{SR}\left(\frac{a_2}{a_3}\right)$$

These equations can be used in conjunction with the normal shock relations previously given or shock tables or the software provided to find the properties of the reflected shock.

Lastly, consider what happens if a gas is flowing out of a duct at a steady rate when the end of the duct is suddenly closed. Since the velocity of the gas in contact with the closed end must again be zero, a shock wave is generated that moves into the moving gas bringing it to rest, i.e., the strength of the shock wave must be such that the velocity reduced to zero behind it. This is illustrated in the following figure.



Moving normal shock wave generated by the closing of the end of a duct out of which a gas is flowing.

A shock wave generated in this way can be analyzed using the same procedure as used to analyze a normal shock wave reflected from a closed end of a duct.

CONCLUDING REMARKS:

A normal shock wave is an extremely thin region at right angles to the flow across which large changes in the flow variables can occur. Although the flow within the shock wave is complex, it was shown that expressions for the overall changes across the shock can be relatively easily derived. It was also shown that entropy considerations indicate that only compressive shock waves, i.e., shock waves across which the pressure increases, can occur and that the flow ahead of the shock must be supersonic. It was also shown that the flow downstream of a normal shock wave is always subsonic. The analysis of normal shock waves that are moving through a gas was also discussed.